23. Ratio, Proportion and Rates of Change

Managing a budget during a period of high inflation.

Given the 2022-23 surge in energy and food prices, your headteacher has asked your class to assist with optimising the school's budget for the following year to absorb these costs. As part of this exercise, you will need to consider how various costs incurred by the school relate to aspects of the school's operations, such as the number of pupils and classrooms.

1. The school is funded on a per pupil basis so the amount of income the school receives (which forms the school's annual budget) is directly proportional to the number of pupils on roll. Express this relationship as a formula.

i = kp (where i = income; k = constant of proportionality; and p = pupils on roll)

2. If the school receives £5,715 per pupil and the school has 900 pupils on roll, what would the school's expected budget be for the year?

£5,717 x 900 = £5,143,500

3. The cost of the school's monthly utility bill is directly proportional to the square root of the number of students. If the school has 900 pupils and has a monthly utility bill of £7,000:

   a. Write an equation to represent the relationship between the number of pupils (p) and the cost of the utility bill (u).

$u = k\sqrt{p}$ (where u= monthly utilities cost and k is the constant of proportionality)

   b. Solve the equation to find the constant of proportionality.

$7{,}000 = k\sqrt{900}$

$k = 7000 \div \sqrt{900}$

$k = 233$

4. The annual insurance premium (P) for a school is directly proportional to the cube root of the school building's value (V). Assuming that the value of the building is £2,000,000 and the annual insurance premium is £15,000, answer the following:

   a. Write this relationship as an equation.

The relationship between the building's value (V) and the annual insurance premium (P) can be represented by the equation:

$P = k\sqrt[3]{V}$

b. Solve the equation to find the constant of proportionality.

£15,000 = k∛£2,000,000

k = £15,000 / ∛£2,000,000

k ≈ 119.055

Therefore, the equation is:

P = 119.055∛V

5. The school has a contract with a stationery company to purchase academic planners for their students. The school pays less per planner if they make a larger purchase. Explain the relationship between the number of planners purchased and the price per planner.

The price per planner is inversely proportional to the number of planners purchased. So the more planners bought the cheaper each unit costs and vice versa.