

## 32. Ratio, Proportion and Rates of Change

Using GDP data to compare economic growth rates across the world.

You are completing an internship at a bank and your first project is to research economic growth in various cities and countries around the world. You learn that economic growth is typically measured in changes in gross domestic product (GDP), which refers to the financial value of goods and services produced by an economy. Additionally, your manager explains that you should also consider GDP per capita, which is calculated by dividing an economy's GDP by its population, in order to better understand the average economic output of each individual person within an economy.

1. The GDP of Sweden was \$496 billion in 2010, and it increased to \$547 billion in 2020. During the same period, the GDP of Israel increased from \$238 billion to \$413 billion.
  - a. By how much did total GDP increase over this period in Sweden and Israel?

GDP increase in Sweden:

$$\$547,000,000,000 - \$496,000,000,000 = \$51,000,000,000$$

GDP increase in Israel:

$$\$413,000,000,000 - \$238,000,000,000 = \$175,000,000,000$$

- b. What are the compound annual growth rates of GDP in Sweden and Israel?

Compound annual growth rate for Sweden = 0.98%

$$496,000,000,000x^{10} = 547,000,000,000$$

$$(547,000,000,000 \div 496,000,000,000)^{1/10}$$

$$x = 1.0098$$

Compound annual growth for Israel = 5.67%

$$238,000,000,000x^{10} = 413,000,000,000$$

$$(413,000,000,000 \div 238,000,000,000)^{1/10}$$

$$x = 1.0567$$

- c. You learn that in 2020 the population of Israel was 10% smaller than the population of Sweden. Which country has the higher GDP per capita?

$$0.9 \times 547 = 492$$

492 > 413 therefore Sweden has a higher GDP per capita.

2. The population of New York City was 8,190,000 in 2010 and by 2022 it had increased to 8,773,000.

- a. If the population of Chicago was 2,697,000 in 2010 and grew at the same rate as New York City between 2010 and 2022, estimate what the population of Chicago was in 2022.

$$8,190,000x^{12} = 8,773,000$$

$$8,773,000 \div 8,190,000 = x^{12}$$

$$x = 1.0057$$

Rate of growth = 0.57% per year

$$2,697,000 \times 1.0057^{12} = 2,887,369$$

- b. If Chicago and New York City each grew at a constant rate between 2010 and 2022, use your answers to Question 2a to estimate the populations of Chicago in 2013 and New York City in 2018.

Population of Chicago in 2013:

$$2,697,000 \times 1.0057^3 = 2,743,382$$

Population of New York in 2018:

$$8,190,000 \times 1.0057^8 = 8,571,000$$

- c. If the GDP of New York City is \$1.5 trillion in 2022, what would the GDP per capita of Chicago need to be in order for Chicago to have the same GDP as New York City in 2022?

$$\$1,500,000,000,000 \div 2,887,369 = \$519,504.08$$

3. You are asked to compare economic growth in Indonesia and the Netherlands. In 2021, both countries had similar sized economies, with Indonesian GDP at \$1.186 trillion and Dutch GDP at \$1.013 trillion. Meanwhile, Indonesian GDP was \$755 billion in 2010. As your team is correcting the 2010 Dutch GDP data you only have the 2015 Dutch GDP figure, which was \$765 billion.

- a. Based on the available data, which country has the higher annual average compound growth rate?

GDP compound growth rate of Indonesia = 4.2%

$$1,186,000,000,000 \div 755,000,000,000 = x^{11}$$

$$x = 1.042$$

GDP compound growth rate of the Netherlands = 4.8%

$$765,000,000,000 x^6 = 1,013,000,000,000$$

$$1,013,000,000,000 \div 765,000,000,000 = x^6$$

$$x = 1.048$$

- b. Is this a fair comparison? Provide reasons for your answer.

No. It is important to keep the time period the same for each country as we are using this data to create an average. If, for example, there was an usual event or period of growth or decline which impacted GDP during these periods, then the average growth rates would be less comparable.

- c. The population of Indonesia was 273.8 million in 2021 and 244 million in 2010. Calculate Indonesian GDP per capita in 2010 and 2021.

In 2021, GDP per capita = 1.186 trillion  $\div$   $x$

In 2010, GDP per capita = 755 billion  $\div$   $y$

4. The GDP of Poland was \$475 billion in 2010 and it increased to \$599 billion in 2020. During the same period, the GDP of the Czech Republic increased from \$209 billion to \$246 billion.

- a. What was the annual compound growth rate of each country's GDP?

Annual compound growth rate for Poland = 2.3%

$$475,000,000,000 x^{10} = 599,000,000,000$$

$$599,000,000,000 \div 475,000,000,000 = x^{10}$$

$$x = 1.023$$

Annual compound growth rate for Czech Republic = 1.6%

$$209,000,000,000 x^{10} = 246,000,000,000$$

$$246,000,000,000 \div 209,000,000,000 = x^{10}$$

$$x = 1.016$$

- b. What was the ratio of the GDP growth rates of Poland to the Czech Republic?

Economic growth of Poland : economic growth of Czech Republic =

$$2.3:1.6 = 23:16$$

- c. The Polish population decreased from 38.1 million in 2010 to 37.9 million in 2020. Comment on how this may have impacted Poland's GDP per capita.

This would increase the GDP per capita of Poland as the population is decreasing while GDP is increasing.