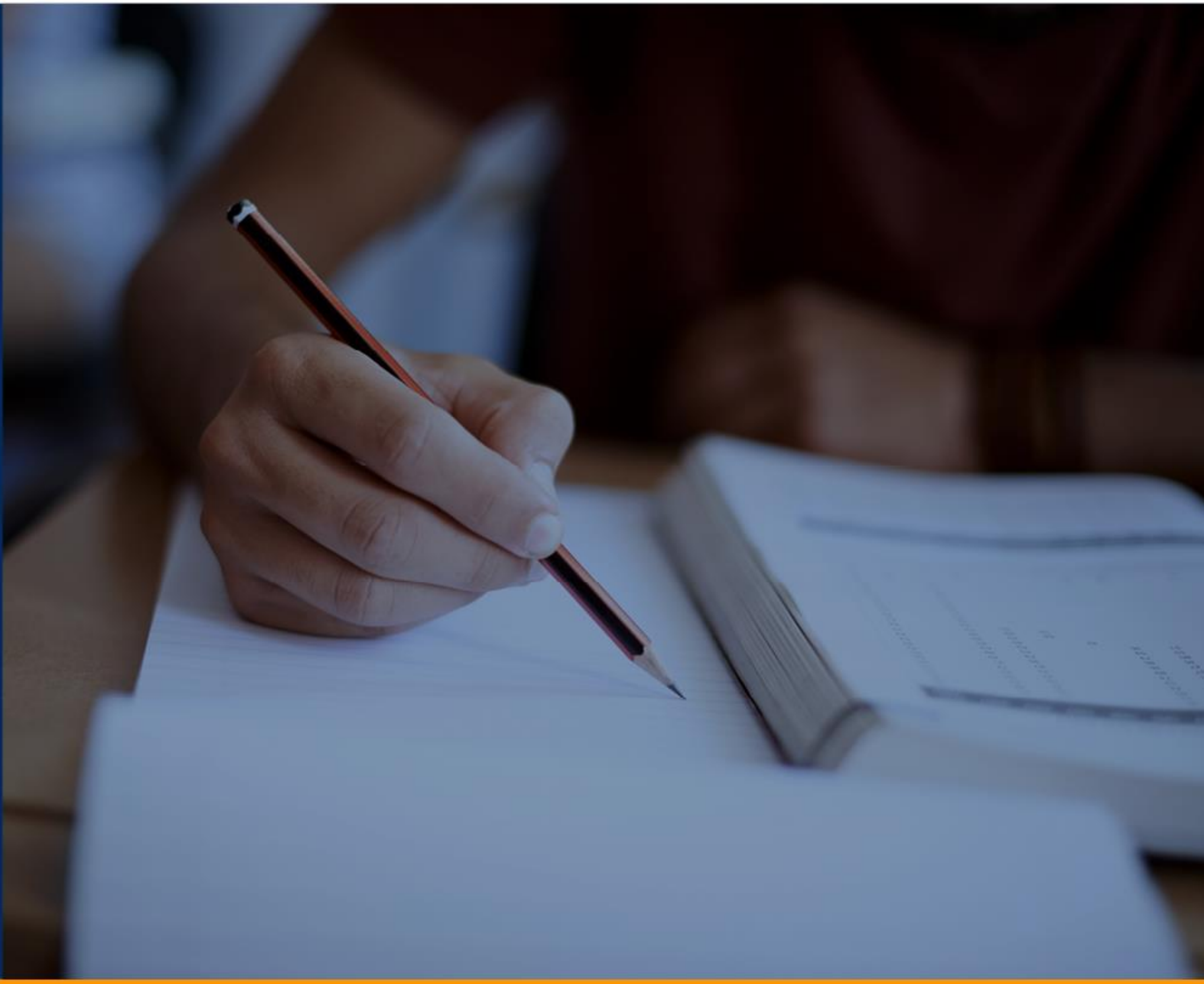
MODELS FOR CALCULATING ILLIQUIDITY

12 November 2020



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CALCULATING ILLIQUIDITY

- Restricted stocks or letter stocks – market based illiquidity discounts
- The Chaffe Model
- Longstaff Lookback Put Option
- Finnerty Average Strike 2012 Model
- Ghaidarov Average Strike Model
- Ghaidarov Forward Starting Model
- Muelbroek CAPM model

RESTRICTED STOCKS OR LETTER STOCKS

- Shares in Listed Companies which cannot be Freely Traded:
  - Shares issued in private placements
  - Trading Between Qualified Institutional Buyers (QIBs)
  - 35% Discounts?
    - Trout – 1968-72 33.5%
    - Moroney ’69-72 35.6%
    - Maher – 1968-73 35.4%

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RESTRICTED STOCKS OR LETTER STOCKS

- The Need for a Safe Harbour:
- Safe Harbour Holding Periods:
- Data Below from VFM Advisory

    - January 1972 – two years: median discount of 22%
    - 1997 – one year: median discount of 16%
    - 2008 – six months: median discount 12%

CHALLENGES WITH THE DATA

- Registration Rights
  - Demand
  - Piggy back
  - Mandatory
- Size of Holdings
  - “Dribble out” rights
    - 1% of stock of company every three months
    - Amount of stock traded in market in previous four weeks
- Volatility of underlying stock

THE CHAFFE MODEL

- We See Further by Standing on the Shoulders of Giants
  - An illiquid stock B, if bundled with a put option, equals liquid stock A
  - Black Scholes Model used:
  - This is a closed form model, with the status of a mathematical proof

THE CHAFFE MODEL (CONTINUED)

The Black Scholes Merton Put Option Formula (including dividends):

$$(Ke^{-rt}\ x\ N(-d_2)) - (Se^{-qt}\ x\ N(-d_1))$$

$$d_1 = \frac{\ln\left(\frac{s}{k}\right) + \left(\left[r - q + \frac{\sigma^2}{2}\right] x\ t\right)}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(\frac{s}{k}\right) + \left(\left[r - q - \frac{\sigma^2}{2}\right] x\ t\right)}{\sigma\sqrt{t}}$$

THE CHAFFE MODEL (CONTINUED)

- The Black Scholes Alternatives:

- $d_2 \ = \ d_1 - \sigma\sqrt{t}$

- $N\ (-d_2) = 1 - N(d_2)$

- $Put\ option = call\ option\ formula + (Ke^{-rt}) - (Se^{-qt})$

THE CHAFFE MODEL (CONTINUED)

- Formula with no dividends and S=K
- $N(d_1) - N(d_2)$
- $d_1 = \dfrac{\left(r+\frac{\sigma^2}{2}\right)x\ t}{\sigma\sqrt{t}}$
- $d_2 = \dfrac{\left(r-\frac{\sigma^2}{2}\right)x\ t}{\sigma\sqrt{t}}$

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THE CHAFFE MODEL (CONTINUED)

- Formula with Risk-Free Rate Set to 0%

- $2N\left(\frac{\sigma\sqrt{t}}{2}\right) - 1$

THE CHAFFE MODEL (CONTINUED)

- The Black Scholes Formula excluding dividends:
- Five Inputs:
  - S Share price
  - K Strike price
  - σ Volatility of share price
  - t Period to exercise in years
  - r Risk free Rate
- Other terms:
  - N Standard cumulative normal distribution function (=norm.s.dist(z, true) in Excel)
  - ln Natural log
  - e exponential number 2.71828….

THE CHAFFE MODEL (CONTINUED)

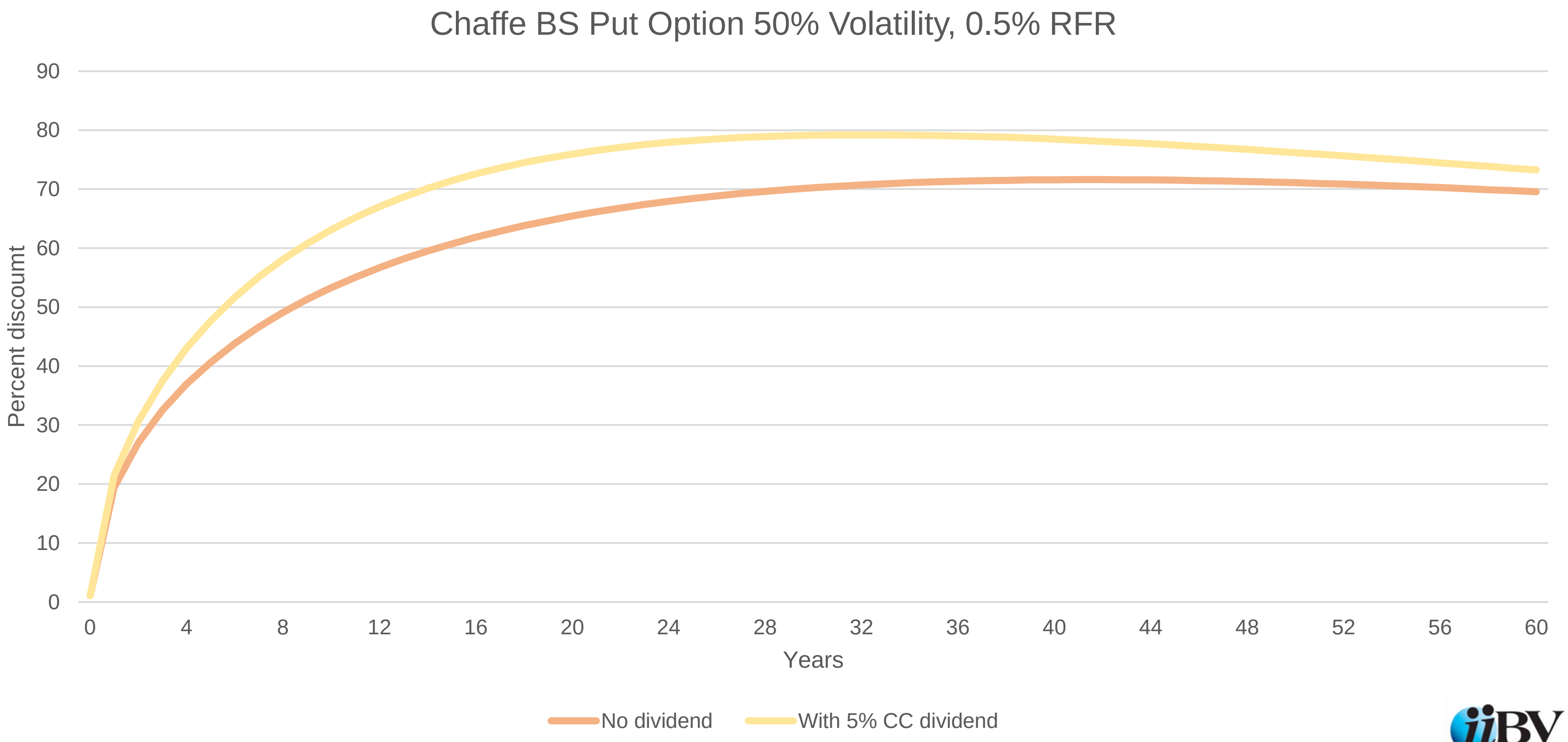
- Black Scholes Merton: RFR 0.5%, volatility 50%.
- Some outputs:
- Indicative DLOM:

	No Dividend	5% dividend
1 year	19.4%	21.5%
5 years	40.6%	47.7%
10 years	53.3%	63.1%
20 years	65.4%	76.0%

THE CHAFFE MODEL (CONTINUED)

- Other Challenges:
  - Why should changes to the risk-free rate impact on the DLOM?
  - Assumption of shareholder fixing the price at start of illiquidity and receiving proceeds at the end.
  - The cost of insuring the opening price is the same for a liquid stock
  - Decline in DLOM for longer periods

THE CHAFFE MODEL (CONTINUED)



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LONGSTAFF LOOKBACK PUT OPTION

- An upper bound on DLOM
- Along the Intellectual Evolutionary Path
- Included for completeness
- Assumes perfect market timing and sale at the highest price in restriction period

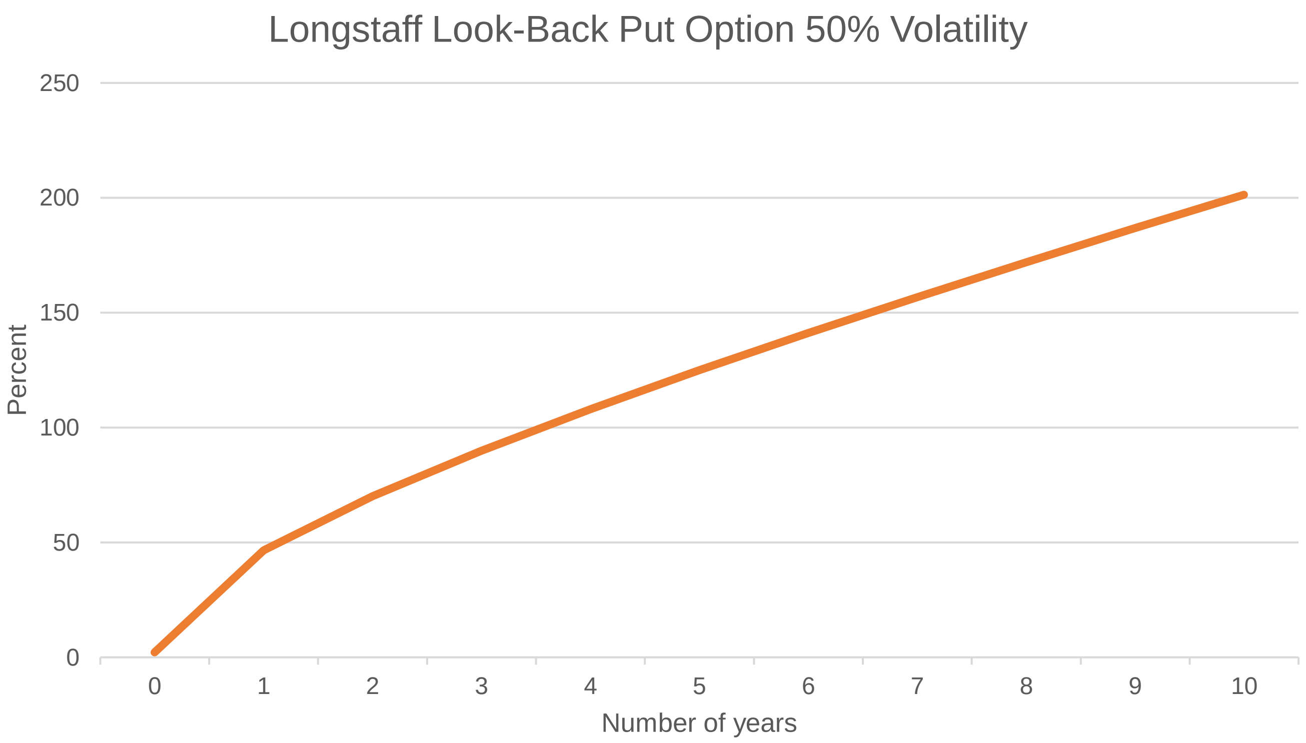
LONGSTAFF LOOKBACK PUT OPTION

- The formula:

- $\left(2 + \frac{\sigma^2 t}{2}\right) \mathrm{N}\left(\frac{\sqrt{\sigma^2 t}}{2}\right) + \sqrt{\frac{\sigma^2 t}{2\pi}} \exp\left(\frac{-\sigma^2 t}{8}\right)$ -1

- Only two inputs:
  - σ sigma – volatility
  - t time period of illiquidity in years

LONGSTAFF LOOKBACK PUT OPTION



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FINNERTY AVERAGE STRIKE 2012 MODEL

- Various versions of the model
- Latest version is the 2012 model
- Estimate of the average price during the illiquidity period
- Assumption that holder will sell at some point in the illiquidity period
- Equal likelihood of sale on any day

FINNERTY AVERAGE STRIKE 2012 MODEL

- The inputs:
  - σ -sigma, the volatility
  - q – the continuously compounding dividend yield
  - t – the time period expressed in years

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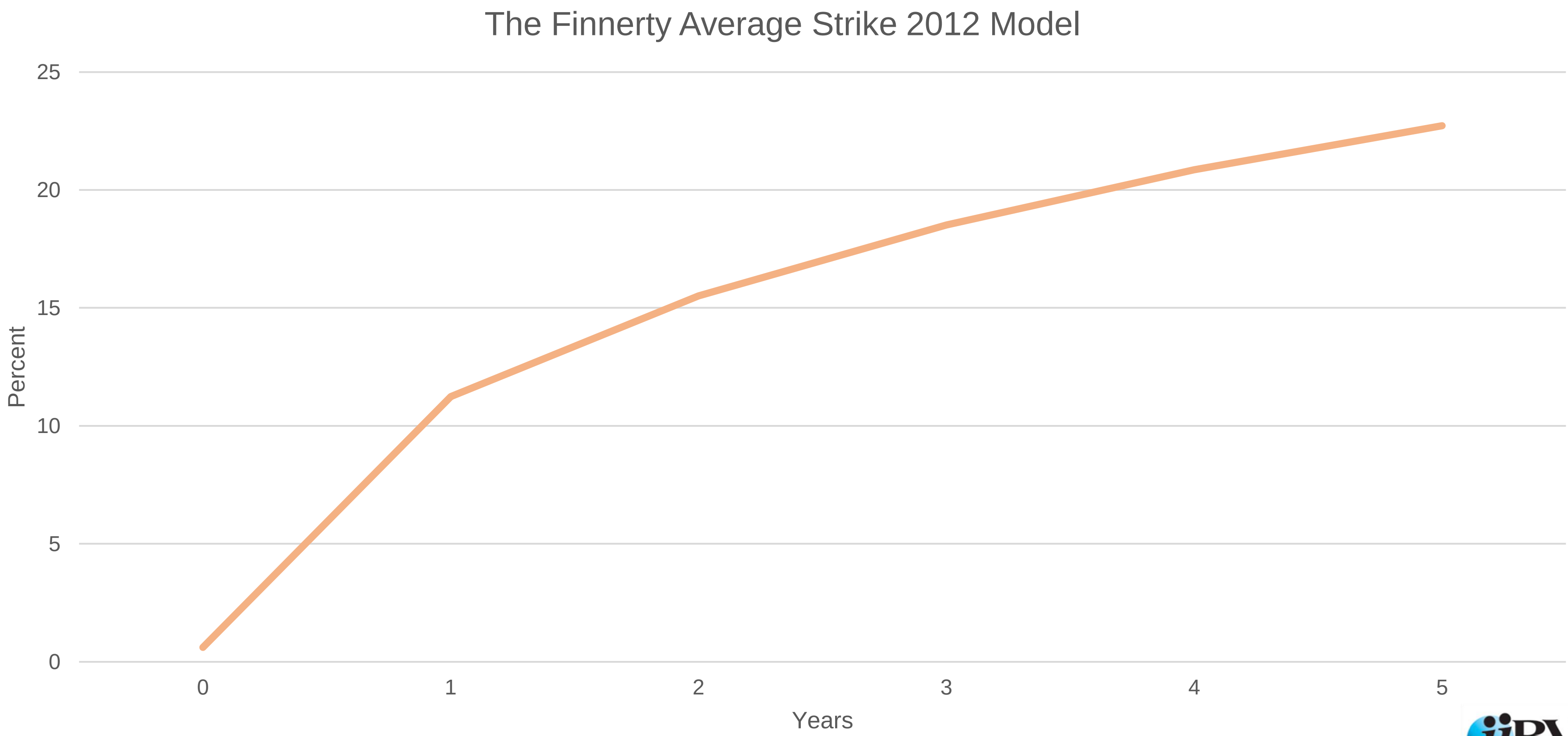
FINNERTY AVERAGE STRIKE 2012 MODEL

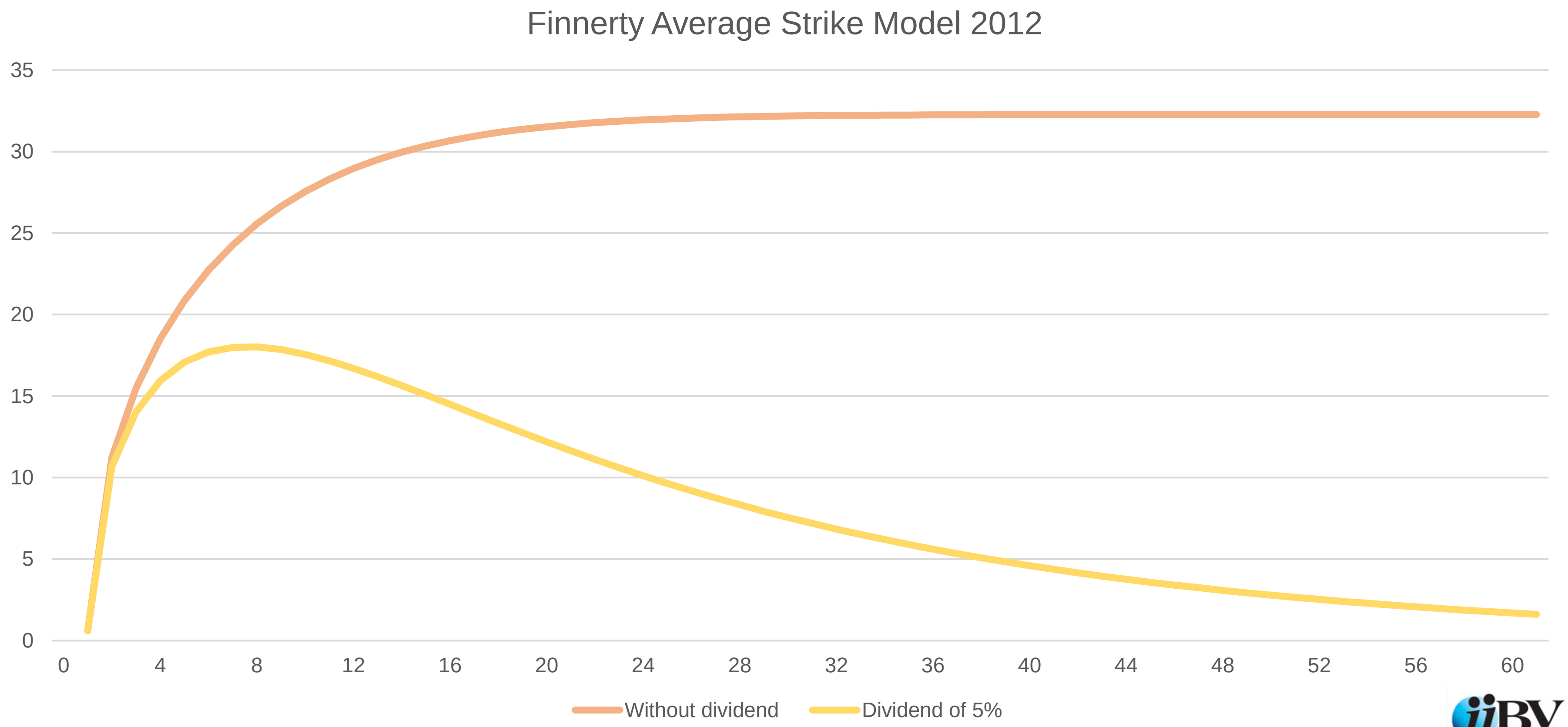
- The Formula:
- $DLOM = V_0 \, x \, e^{-qt} \left(N\left(\frac{v\sqrt{t}}{2}\right) - N\left(-\frac{v\sqrt{t}}{2}\right) \right)$
- $DLOM = V_0 \, x \, e^{-qt} \left(2 \, x \, N\left[\frac{v\sqrt{t}}{2}\right] - 1 \right)$

- $v^2 t = \sigma^2 t + ln\left[2 \, x\left(e^{\sigma^2 t} - \sigma^2 t - 1\right)\right] - \left(2ln\left[e^{\sigma^2 t} - 1\right]\right)$
- $v\sqrt{t} = \sqrt{v^2 t}$
- $-N\left(-\frac{v\sqrt{t}}{2}\right) = +N\left(\frac{v\sqrt{t}}{2}\right) - 1$

FINNERTY AVERAGE STRIKE 2012 MODEL



FINNERTY AVERAGE STRIKE 2012 MODEL



DIVIDENDS IN THE MODELS

- The Problem with e^{-qt}
- Dividends Treated as an Annuity
- Increasing Proportion of Share Value Carved out as Dividend
- DLOM calculated on the balance

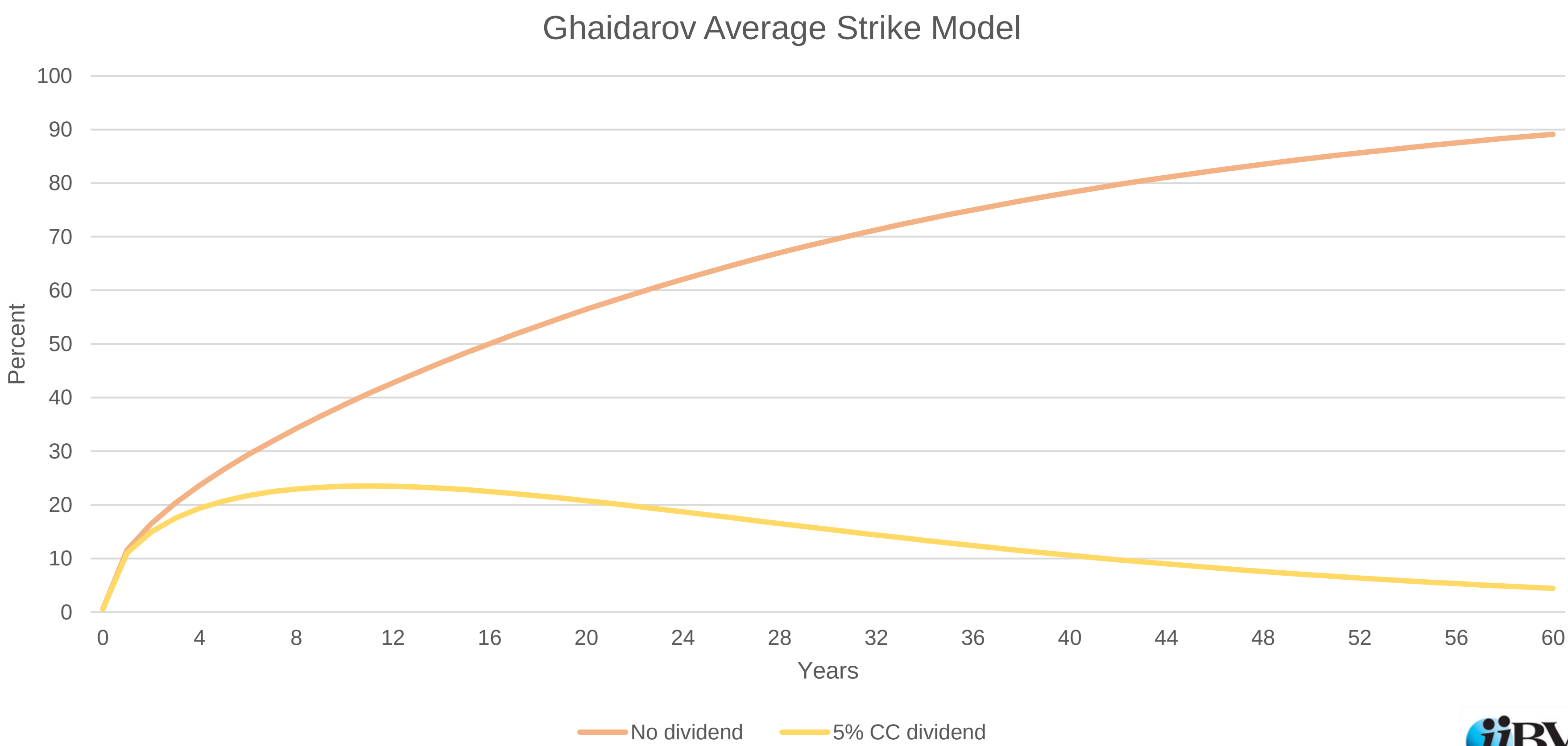
GHAIDAROV AVERAGE STRIKE MODEL

- Response to Finnerty Model in 2009
- An alternative approximation for the average value.

GHAIDAROV AVERAGE STRIKE MODEL

- The Formula:
- $DLOM = V_0 \; x \; e^{-qt} \left(2 \; x \; N \left[\frac{v\sqrt{t}}{2} \right] - 1 \right)$ [Finnerty and Ghaidarov]
- $v^2 t = \ln\left[2 \; x \left(e^{\sigma^2 t} - \sigma^2 t - 1\right)\right] - (2 \; x \; ln[\sigma^2 t])$ [Ghaidarov]
- $v^2 t = \sigma^2 t + ln\left[2 \; x\left(e^{\sigma^2 t} - \sigma^2 t - 1\right)\right] - \left(2 \; x \; ln\left[e^{\sigma^2 t} - 1\right]\right)$ [Finnerty]

GHAIDAROV AVERAGE STRIKE MODEL



GHAIDAROV FORWARD STARTING MODEL

- Liquidity represents a lack of flexibility
- Discounts for lack of marketability should not contain a form of insurance policy
- Forward starting option – bought at the start but strike price is selected at any time

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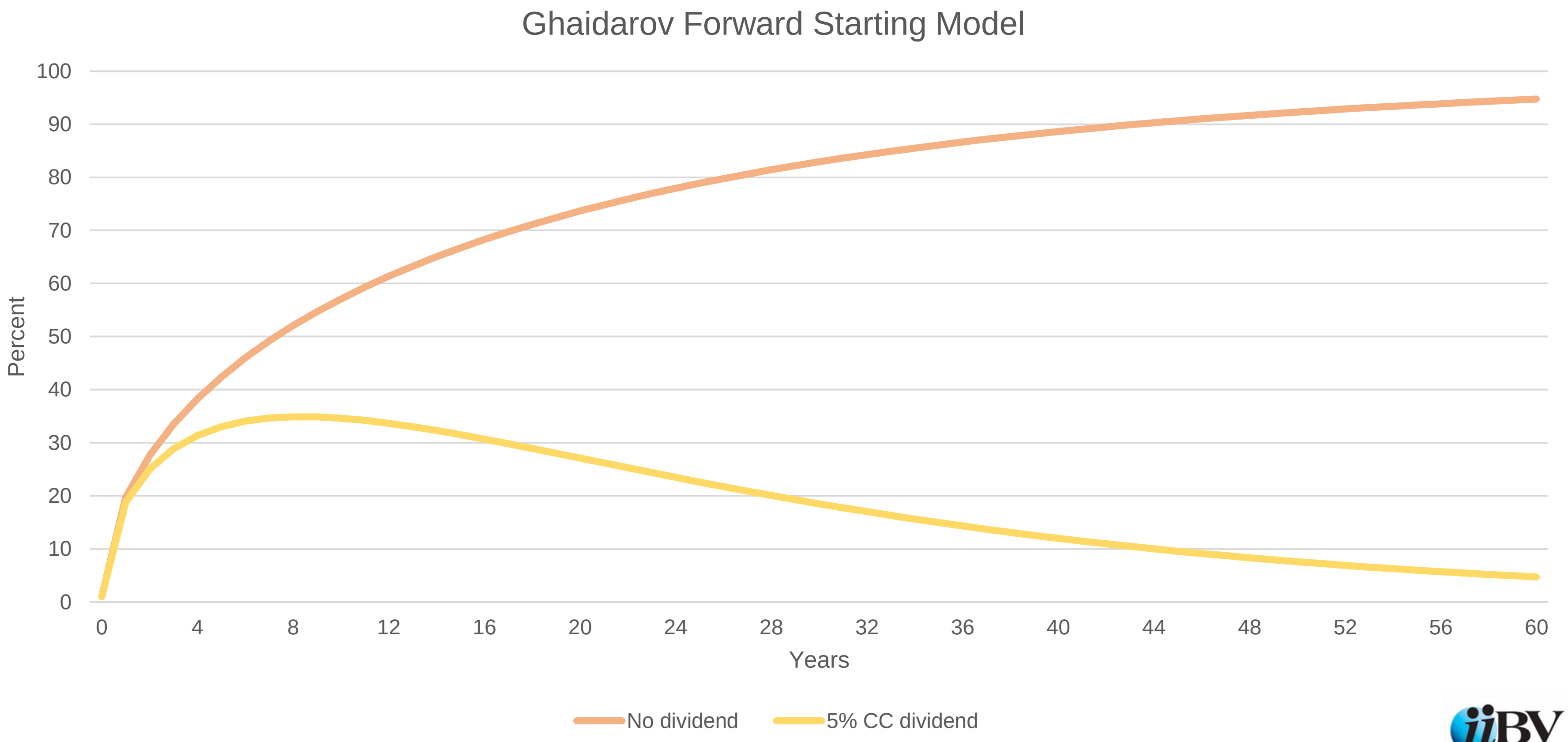
GHAIDAROV FORWARD STARTING MODEL

- A Closed Form Equation

  - $e^{-qt}x\left(2\ x\ N\left[\frac{\sigma\sqrt{t}}{2}\right]-1\right)$

GHAIDAROV FORWARD STARTING MODEL



THE PROBLEM OF DIVIDENDS

Proposal From Ghaidarov:

Treat them in the model as shortening the illiquidity period?

Example: a share is illiquid for 24 months and pays out a 5% dividend at 6 months and at 18 months:

> 5% as share with 6 months’ illiquidity
> 4.75% as share with 18 months’ illiquidity
> 90.25% as share with 24 month’s illiquidity

Equivalent period of illiquidity is 1.9 years

MUELBROEK CAPM MODEL

- Muelbroek CAPM Model
- Discounting to Net Present Value
- Based on Beta and Total Beta

  - Capital asset pricing line:
  - $R_p = R_f + \frac{\sigma_p}{\sigma_m}\, x \left(R_m - R_f\right)$
  - $Beta = \frac{\sigma_p}{\sigma_m}\, x\ correlation\ with\ the\ market$
  - $Total\ Beta = \frac{\sigma_p}{\sigma_m}$

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MUELBROEK CAPM MODEL

- The Formula:
  - $DLOM = 1 - \frac{1}{(1+R\,)^n}$
  - $R = ERP\ x\left(\frac{\sigma_s}{\sigma_m} - Beta\right)$
  - $R = ERP\ x(Total\ Beta - Beta)$
  - $ERP = market\ equity\ risk\ premium$
  - $\sigma_s = volatility\ of\ the\ shares\ of\ the\ company$
  - $\sigma_m = volatility\ of\ the\ market$

MUELBROEK CAPM MODEL

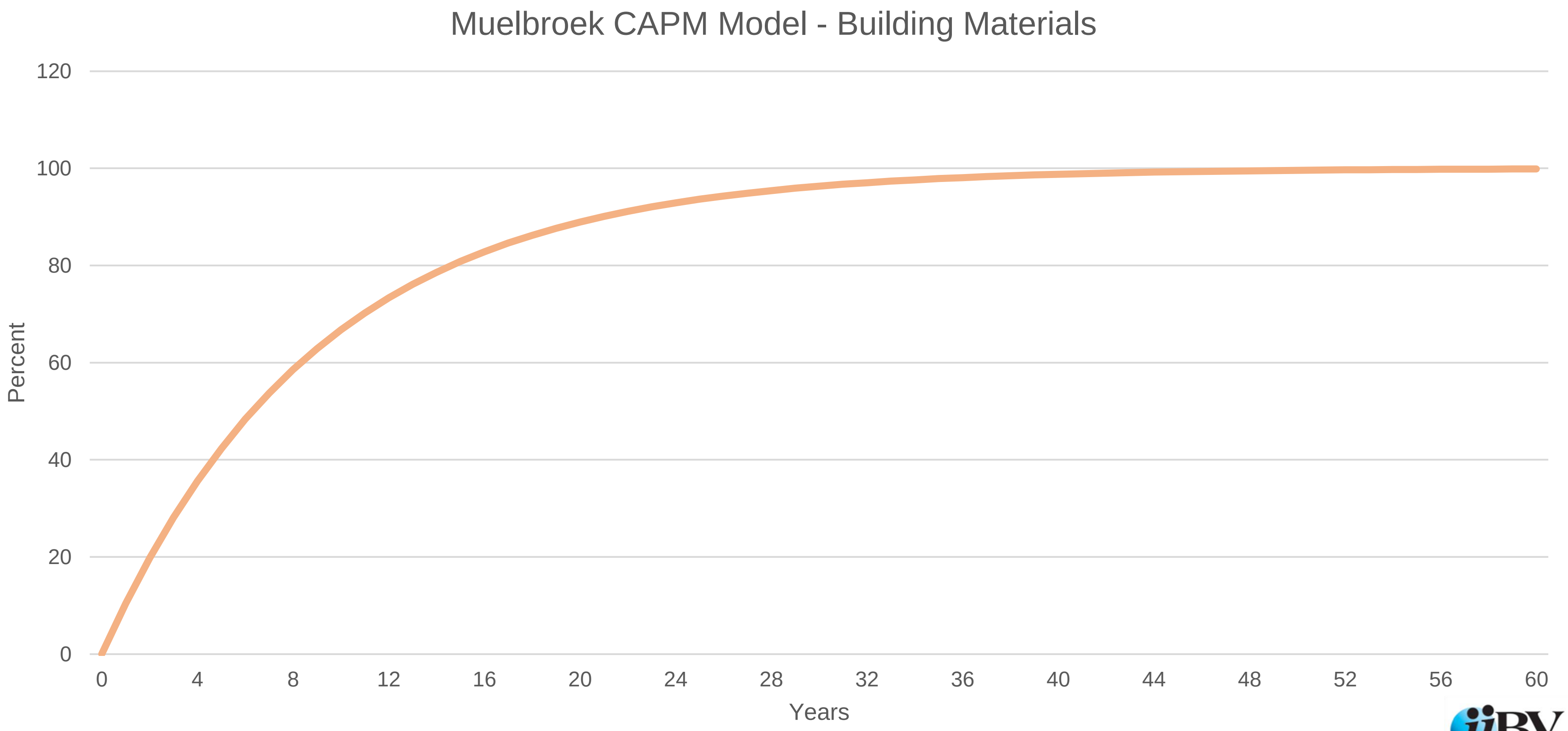
Application in Practice

Data for Beta and Total Beta from Damodaran – January 2020

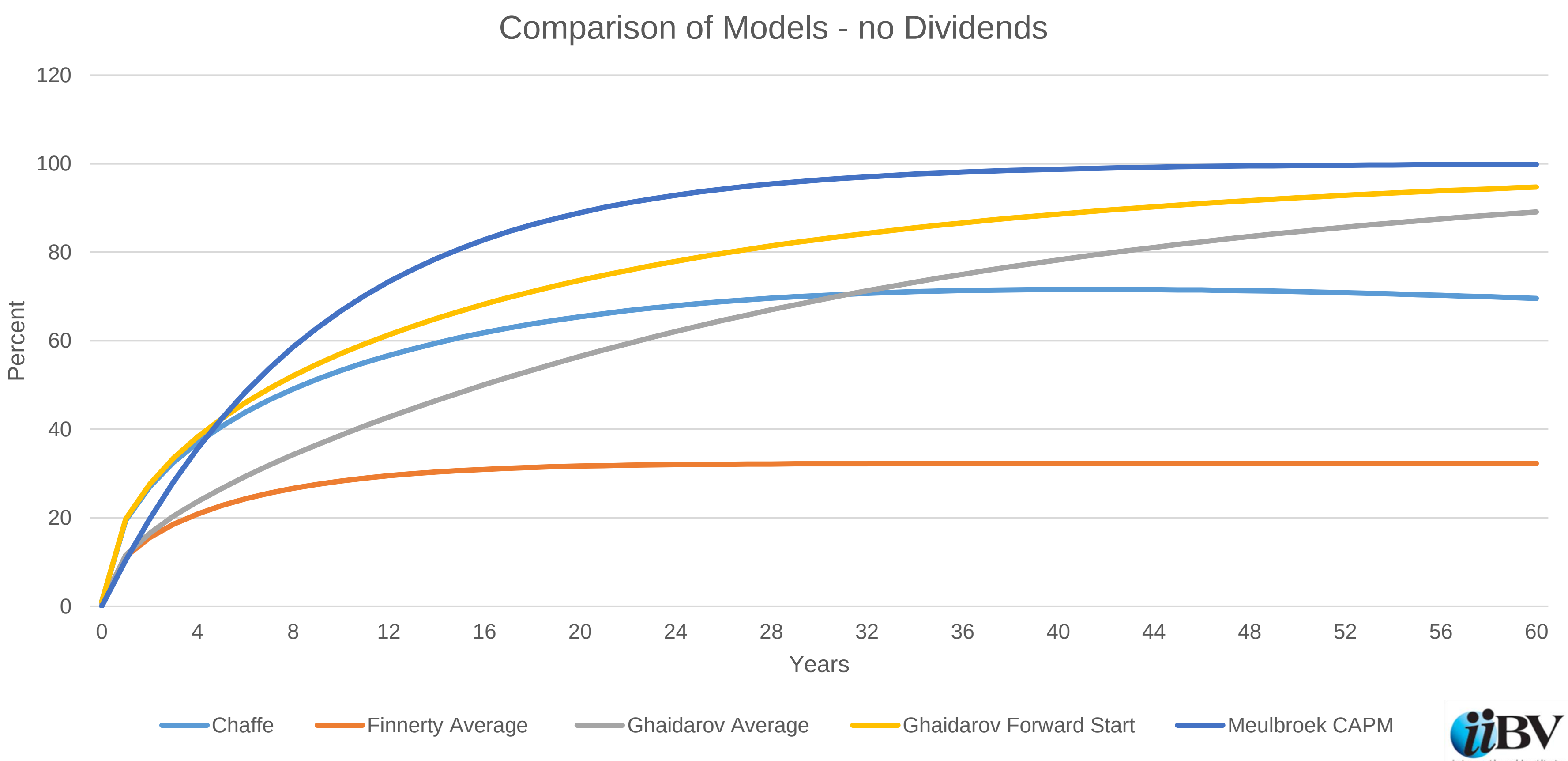
Building Materials Sector

Beta	1.23
Correlation with market	38.8%
Therefore total Beta	3.17

MUELBROEK CAPM MODEL



COMPARING RESULTS – NO DIVIDENDS



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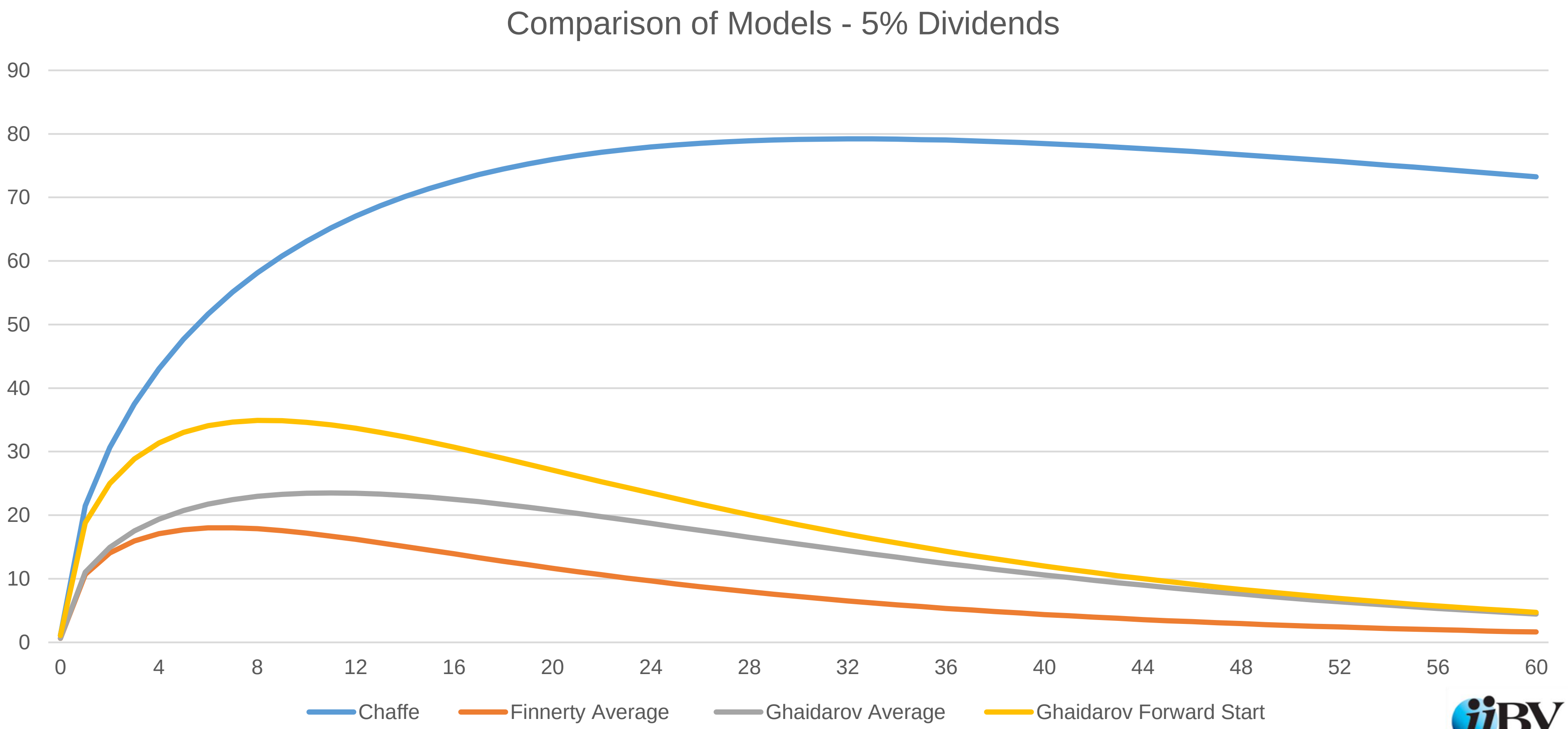
COMPARING RESULTS – 5% DIVIDENDS



SOME CONCLUSIONS

- The various models reflect the evolutionary development of ideas
- The Ghaidarov Forward Starting Model – Liquidity as a loss of Choice – back to Black Scholes
- Muelbroek Model – a means of introducing Total Beta into valuation via DLOM
- Response to Dividends – shorten deemed period of illiquidity

AND FINALLY…

Thank you for listening!

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